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LABORATORY FOR INFORMATION AND DECISION SYSTEMS
Massachusetts Institute of Technology
Cambridge, Massachusetts 02139

Final Report for
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PROBABILISTIC MODELING AND STATISTICAL INFERENCE FOR RANDOM FIELDS AND SPACE-TIME PROCESSES

Submitted to: Dr. Julia Abrahams
Code 311
Room 607
Mathematical Sciences Division
Office of Naval Research
800 North Quincy Street
Arlington, VA 22217-5660

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1. INTRODUCTION

In this report we summarize the research effort, funded under ONR Grant N00014-91-J-1004, on probabilistic modeling and statistical inference for random fields and space-time processes. The research we have pursued in this project consists of the investigation of a set of interrelated topics involving the development of new mathematical methods for the challenging problems of random field analysis and inference and the application of these methods to problems of practical significance. In particular, the problems of interest in this work were motivated by and directly address issues that are central both to the challenging large-scale data assimilation and estimation problems arising in physical oceanography and to a number of other remote sensing and imaging problems of direct interest to the Navy.

During the grant period we have had what we feel is considerable success in our research efforts. In particular the research supported in part by our ONR grant has led to 90 publications, including 40 journal papers, 36 conference papers and presentations, 2 book chapters, 3 SM theses, and 9 Ph.D. theses. A number of additional papers and theses at both the SM and Ph.D. level are currently in progress. Furthermore, our research in this area has received considerable national and international recognition. In particular, Professor Willsky has given several keynote and plenary lectures on research topics supported by our ONR grant. These include:

•University of Southern California Signal and Image Processing Institute Distinguished Lecture, Feb. 1991.

•Featured invited lecture at the SIAM Conference on Linear Algebra and Its Applications, Sept. 1991.

•Keynote address at the IEEE International Conference on Systems Engineering, Aug. 1991.

•Principal lecture on Multiresolution Methods in Image Analysis at the Tri-Service Workshop on Statistical Methods in Image Processing, May 1992.

•Keynote address and principal lectures at the inaugural workshop for the Centre for Robust and Adaptive Systems", Canberra, Australia, February, 1992, involving both researchers and members of the Australian government.

•Featured invited lecture on challenges in signal processing at the SIAM Workshop on Systems and Control, September 1992.

•Keynote address on challenges in signal processing at the Twenty-Fifth Anniversary Celebration for INRIA (Institut National de Recherche en Informatique et en Automatique), held at the Ministry of Research, Paris, France, December 1992.

•Plenary address at the 1993 IEEE Symposium on Image and Multidimensional Signal Processing, Cannes, France, September 1993.

•Principal invited lecture on statistical problems in image analysis and random fields at The Joint Statistical Meetings of The American Statistical Association, the Biometric Society, the Statistical Society of Canada, and the Institute of Mathematical Statistics, Toronto, August 1994.

In addition, Mr. Paul Fieguth, a Ph.D. student with Prof. Willsky has had several honors, including the Student Paper Award at the 1994 IEEE Oceans '94 Meeting in Brest, France and invited lectures at the 1994 International Conference on Image Processing in Austin Texas and at the upcoming 1995 International Conference on Acoustics, Speech, and Signal Processing.

Furthermore, during this time period we have had considerable interaction with Navy personnel and with problems of direct Navy interest, leading both to the application and transitioning of our research and to the formulation and study of a variety of research problems that have been inspired by discussions with Navy personnel or that have been motivated by Navy missions and needs:

•Our initial research on multiresolution methods for estimating velocity fields in space-time data and sequential imagery has been applied by Alphatech, Inc. to the problem of image-based tracking of targets under an SBIR (Small Business Innovative Research) Phase I project sponsored by NAVAIR and directly monitored and supported by Dr. Gary Hewer at NWC China Lake. The continuation of this work under a Phase II program is currently under consideration, with the endorsement of Capt. Obar, PMA for the Sidewinder Program.

•The framework we have developed for modeling random fields at multiple resolutions and in particular for estimation and detection in fractal-like backgrounds is also being transitioned by Alphatech in the context of detecting small targets, such as periscopes, from P-3 SAR and ISAR imagery, under both a second NAVAIR SBIR and directly with NWC (Dr. Oran McNiel).

•Prof. Willsky has had direct contact with researchers and engineers at the Naval Coastal Systems Station (including Drs. E. Moritz, G. Dobeck, and A. Dubey) involved in the development of multisensor, multispectral methods for imaging and automatic detection and recognition of mines. This interaction began with discussions at the 1992 Tri-Service Workshop mentioned previously and has continued with two trips that Prof. Willsky has taken to Panama City in 1993, the first at the invitation of Dr. Dubey (during which Prof. Willsky gave a lecture on his work on multiresolution image analysis) and the second at the ONR/CSS Workshop held in November 1993.

•In addition, Prof. Willsky has interacted with researchers at The Charles Stark Draper Laboratory who are working under contract to CSS in this area. Indeed, at this time Draper and Dr. Dobeck are discussing the incorporation of some of our ideas for the analysis of synthetic aperture sonar data collected at CSS.

•The algorithms that we have developed for multiresolution sensor data analysis are also currently being transitioned by Draper Lab in its work on SAR image analysis.

•One of Prof. Willsky's former graduate students, Dr. T.M. Chin, has been working with Prof. A. Mariano at The University of Miami on the problem of spatio-temporal tracking of the North Wall of The Gulf Stream using methods of optimal estimation developed in part by Dr. Chin during his graduate work at MIT under our ONR Grant. The problem of mapping and tracking North Wall position is of considerable interest to the Navy and in fact such maps are regularly produced by NRL researchers at the Stennis facility in Mississippi.

•We have recently demonstrated the power of our multiresolution random field estimation framework in the context of processing satellite altimetry data from the TOPEX/Poseidon satellite for the estimation of ocean height perturbations in the North Pacific. Such estimates are of critical importance in ocean science programs funded by the Navy and by other national scientific organizations, as they provide inputs to global ocean circulation models currently under development and study.

•Ms. Lori Belcastro, another of Prof. Willsky's graduate students, is the recipient of an ONR Graduate Fellowship.

A complete list of the papers arising out of our research is included at the end of this report. In the following sections we provide a brief summary of the most recent of these results.

2, MULTIRESOLUTION MODELING, ESTIMATION, AND STATISTICAL INFERENCE

This area, which represents the theoretical basis for much of our work on random fields, covers our continuing efforts in developing and exploiting the framework we have developed for multiresolution modeling, estimation, and statistical inference for random fields. Our most recent publications in this area are [58-60, 62, 71, 78, 80, 86]. In particular, the results reported in [58] provide us with the multiscale statistical characterization of the error in the estimation of a random field based on measurement data that may be sparse, irregularly sampled, multiresolution, etc. The significance of this result is twofold. First, it allows us to compute error statistics for random field estimation problems of considerable size (e.g. 1000 x 1000 random fields), something that has heretofore been impossible. Secondly, it provides us with one of the key elements needed for efficient space-time estimation of large-scale random fields, since the key step in such a problem is the use of new data to estimate the error in the estimate based on preceding data sets. These results will be of critical importance in the continuation of our research on space-time data assimilation.

The results reported in [71, 78, 80, 86] are of importance as they provide the statistical basis for building multiresolution models from data or from covariance estimates of a random field. Given the substantial advantages of our multiresolution modeling framework, the key questions are in identifying under what conditions it is applicable--i.e., when the random fields of interest can be efficiently modeled within our framework--and in developing methods to construct these models. Of particular interest here are the results in [80] and [86]. In particular, the method described in [86] provide an extremely powerful methodology for building multiresolution models that have desirable properties, such as appropriate levels of smoothness, for a surprisingly large class of random fields including but not limited to those that are typically described using Markov Random Fields (MRFs). The work in [80] demonstrate the promise of these modeling methods by showing their applicability in modeling SAR imagery and in using the resulting models in discrimination problems.

3. ESTIMATION OF GEOMETRIC FEATURES IN RANDOM FIELDS

The second area of our research focuses on the estimation of geometric features of random fields. Our most recent work in this area is described in [54-57, 61, 63, 64, 89]. The research described in [54-57, 61, 89] deals with the direct estimation of geometric features from tomographic measurement data. These results, and in particular those in [54], provide the basis for a new approach to tomographic reconstruction from extremely

limited data, as is typical in oceanacoustic tomography. In particular, our results indicate how we can use tomographic data to estimate the spatial moments of a random field which in turn can be used together with a nonlinear, divergence-based regularization criterion to enhance reconstructions obtained using standard tomographic techniques. Thus, for example, if we have a priori information about the location, size, or shape of spatial features--such as cold- or warm-core eddies--we can use this together with the estimated moments to enhance reconstruction in the vicinity of the eddy. These same methods are also of potential value for nondestructive evaluation applications in which sparse tomographic data are to be used to detect and characterize flaws, cracks, and corrosion. The work in [63, 64] deals with the problem of tracking the geometry of objects embedded in random fields that change with time.

4. INVERSE PROBLEMS

The third component of our research deals with multiresolution methods for inverse problems, and our recent work in this area is described in [47-49, 65, 66, 68-70, 72, 76-77, 81-84, 90] In particular, the research in [48-49, 65, 68, 72, 76, 84, 90] deals with the development of multiresolution methods for tomographic reconstruction that (a) are extremely efficient; (b) allow us to tradeoff resolution and accuracy in a statistically meaningful way; (c) provide direct imaging of features at a hierarchy of resolutions, allowing us to identify significant anomalies; and (d) provide a new and computationally superior method for direct texture discrimination from tomographic data.

Our work in [47, 66, 69, 70, 77, 81-83] deals with the development of multiresolution inversion methods for a variety of inverse problems, including those specified by the partial differential equations arising in inverse scattering problems. These results have many of the same characteristics as our results on tomographic reconstruction, with some important differences in emphasis and in technique due to the greater complexity of the Green's functions characterizing inverse scattering problems. In particular, in this context we have developed a precise statistical criterion for determining the optimal scale of reconstruction at each point in an image and for assessing which measurements in a set of scattering experiments provide useful information for reconstruction at each point in the region being imaged. In this way we completely expose the structure of multisensor fusion for such problems. In addition, we have now also developed a coarse-to-fine approach to detecting and isolating anomalous regions in a medium from inverse scattering measurements [81], allowing us, in a data-adaptive way, to "zoom" in on regions of interest and to direct the limited numbers of degrees of freedom in our reconstruction into areas in which additional detail is present and can be estimated. In addition, the inverse problems of inverse scattering are fundamentally nonlinear, as are many inversion problems (including oceanacoustic tomography). In much of our work in this area we focused on developing inversion methods based on linearization techniques (most notably the so-called Born linearization method),which are frequently quite accurate (if the deviations from the nominal background represent "weak" scattering perturbations). In our most recent work [83] we have begun to take a careful look at the fully nonlinear problem. In particular, we have developed results on the sensitivity of linearized inversions to model errors induced by nonlinear effects. One of the novel aspects of this work is that our formalism for determining the optimal scale for reconstruction allows us to determine the required resolution to which we must accurately capture nonlinearities. For example, if the scale to which statistically significant reconstruction is possible is coarser than the scale at which nonlinear effects become significant, then the use of a linearized technique is sufficient.

5. APPLICATIONS OF MULTIRESOLUTION STATISTICAL METHODS IN OCEANOGRAPHY AND COMPUTER VISION

Finally, the fourth segment of our recent research [67, 73-75, 79, 85-88] deals with the application of our multiresolution methodology for estimating random fields to several problems of considerable practical importance. In particular, the estimation of ocean height patterns is an essential element in understanding ocean circulation, as variations in sea level provide indications of pressure differentials that drive the circulation. Maps of sea level variations are often used as an end result in themselves, providing images of circulation patterns--roughly speaking the ocean equivalent of atmospheric highs and lows--and changes in those patterns in time. In addition, these maps are of considerable value to ocean modelers. In particular, global circulation modelers, such as our colleagues at MIT, use such sea level variations both to drive their models and to correct for errors between the predictions provided by their simulation models and the estimates of ocean height based on measurements. Roughly speaking, we can think of this as an extremely large time-recursive distributed parameter estimation problem, in which a global circulation model is used to capture ocean dynamics and in which maps of sea level variations are used as the measurements. In order for this to be done in a statistically meaningful way, of course we must have a measure of the quality of such maps. That is, we require knowledge of the covariance of the errors in our estimate of ocean height. Such covariance estimates are of use for other reasons as well, as they allow us to determine if particular features in a map are statistically significant or not, a point on which we will comment further in a moment.

There are a number of significant challenges in producing such maps of sea level variations and their associated error statistics. The first is the fact that the data on which such estimates are to be based are sparse and irregularly sampled, s is certainly the case for the TOPEX/Poseidon satellite altimetry data that we have used in our work. The irregularity of the data pattern presents a major challenge as there is no regular structure that can be used to advantage, thus, for example, precluding the direct use of efficient Fourier transform techniques. Furthermore, there are other sources of nonstationarity not only in the expected variability in sea level in different portions of the ocean but also in the quality of the measurements provided by the satellite. In particular, the satellite altimeter provides direct measurements of the distance from the ocean surface to the satellite, while what is desired is a measurement of ocean height relative to the geoid (the equipotential surface of the earth's gravity field). Since the geoid is not known perfectly, especially when one accounts for the presence of variations in ocean floor height due to underwater trenches, mountain chains, sea mounts, etc., errors in the geoid translate directly into errors in the sea level estimates produced by the "corrected" altimetry measurements. Thus, the resulting measurement "noise" in the altimetry data is spatially nonstationary.

In addition to having to deal with irregular data and nonstationary phenomena, we also must deal with the fact that the set of variables to be estimated is extremely large. For example, in our work we have focused on estimation in the North Pacific, a region consisting of 300,000 - 500,000 grid cells representing the finest scale at which we may produce estimates. Thus in principle, producing a map of sea level variations over this region corresponds to a 300,000 - 500,000-dimensional estimation problem--and on top of that we also want the error covariance for this estimated map! To our knowledge and that of our oceanographer colleagues, no methods developed by others are capable of dealing with problems of this size. In contrast, as we have shown in [67, 73-75, 79, 87], using our methods we can perform these computations with considerable ease, allowing us not only to provide the maps and error statistics mentioned previously but also to investigate a variety of related problems of considerable importance in oceanography.

In particular, in our initial studies we have used our multiresolution estimation methods based on a so-called "fractal" prior model--i.e., a 1/f-like spectrum--that is consistent both with the statistics of the satellite data and with the form of the models proposed by oceanographers. Such models capture spatial structure over all scales and thus are well-matched to the multiscale nature of oceanic phenomena. In addition, as we have pointed out, the accuracy of the satellite measurements is spatially varying, thanks to the varying levels of uncertainty in the geoid correction that has been performed on the satellite data. We have captured this in our work by the use of a spatially-varying measurement noise variance developed in consultation with our oceanography colleagues and based on a careful examination of the nature of geoid errors.

The results of our work in this area have been extremely encouraging and have already caught the attention of the oceanography community, both at MIT and internationally (as evidenced by Mr. Fieguth's paper award and the excellent reception our work has received). In particular, our method allows us to produce maps of sea level variations at far finer scales than those that oceanographers had previously used. In addition, as we have stated previously, one of the major challenges in statistical inference and probabilistic data assimilation for oceanographic analysis is the enormous size of the problems that must be considered. Indeed, using standard techniques, such as Markov random fields, computing the estimates for the ocean region considered would be an exceedingly difficult computational problem, and the calculation of the statistical characteristics of the estimation errors would be prohibitively complex. Using our methods, the processing of 10 days' worth of satellite data over the North Pacific, to produce an estimated field at multiple resolutions, together with full error covariance information takes less than 1 minute on a Sparc 10 workstation.

Thus not only can we solve problems such as the ones that we have just described which had previously been considered prohibitively complex, but we can also consider a variety of deeper and more ambitious questions of considerable interest to the Navy and the oceanographic community. In particular, we have used the error covariance calculations to allow us to identify areas in which the satellite data are anomalous and have verified that indeed these outliers correlate remarkably well with regions in which there are significant bathymetric features and geoid uncertainty. In addition, by using a higher-order version of our algorithm, we have been able to consider the joint estimation of sea level and geoid errors, yielding enhanced maps of the geoid which are of potential value in a wide variety of contexts.

Moreover, inspired by our success in oceanography and by the fact that oceanographers are actually most interested in the gradient of the ocean surface (as this provides information on currents and pressure fields), we have developed new general methods for surface and surface gradient reconstruction from noisy and possibly sparse data [85, 88]. These results are not only of interest in oceanography but also are of independent interest in computer vision, as they provide new methods for surface reconstruction that overcome many of the same problems that had confronted researchers in oceanography, namely computational complexity and the ability to calculate error statistics.

Finally, one of the other strengths of these multiresolution models is that they also allow us to calculate likelihood functions for our models, permitting us to both validate models and estimate parameters. Applications of this capability in both oceanography and computer vision are given in the references cited.

PUBLICATIONS

The following is a list of papers and theses describing research supported in part by ONR Grant N00014-91-J-1004.

1. R. Nikoukhah, A.S. Willsky, and B.C. Levy, "Kalman Filtering and Riccati Equations for Descriptor Systems", *IEEE Transactions on Automatic Control*, Vol. 37, No. 9, Sept. 1992, pp. 1325-1342.

2. R. Nikoukhah, D. Taylor, B.C. Levy, and A.S. Willsky, "Graph Structure and Recursive Estimation of Noisy Linear Relations," accepted for publication in *Journal of Mathematical Systems, Estimation, and Control.*

3. D. Taylor and A.S. Willsky, "Maximum Likelihood Estimation for Two-Point Boundary-Value Descriptor Systems", Johns Hopkins Conf., Baltimore, March 1991.

4. D. Taylor and A.S. Willsky, "Parallel Smoothing for Causal and Acausal Systems," Proc. IEEE Conf. on Dec. and Control, Brighton, England, Dec. 1991.

5. D. Taylor and A.S. Willsky, "Inward-Outward Recursions, Domain Decomposition, and Parallel Algorithms for Optimal Multidimensional Signal Processing," IEEE Workshop on Multidimensional Signal Processing, Lake Placid, NY, Sept. 1991.

6. D. Taylor, "Parallel Estimation for Noncausal Processors", Ph.D. thesis, January 1992.

7. D. Taylor and A.S. Willsky, "Parallel Estimation in One Dimension," journal paper in preparation.

8. D. Taylor and A.S. Willsky, "Inward-Outward Recursions, Domain Decomposition, and Parallel Algorithms for Optimal Multidimensional Estimation," journal paper in preparation.

9. M.M. Daniel, "Parallel Algorithms for 2-D Boundary Value Systems," SM Thesis, MIT Dept. of EECS, Feb. 1993.

10. M. Daniel and A.S. Willsky, M.M. Daniel and A.S. Willsky, "Efficient Algorithms for Two-Dimensional IIR Filtering," submitted to *Signal Processing.* .

11. T.M. Chin, W.C. Karl, and A.S. Willsky, "Sequential Filtering for Multi-Frame Visual Reconstruction," *Signal Processing,* Vol. 28, Aug. 1992, pp. 311-333.

12. T.M. Chin, W.C. Karl, and A.S. Willsky, "Sequential Optical Flow Estimation Using Temporal Coherence," *IEEE Trans. on Image Processing.*

13. T.M. Chin, "Dynamic Estimation in Computational Vision", Ph.D. thesis, October 1991.

14. T.M. Chin and A.S. Willsky, "A Sequential Filter for Multi-Frame Visual Reconstruction: Computing Optical Flow with Temporal Coherence", 7th IEEE Workshop on Multidimensional Signal Processing, Lake Placid, September 1991.

15. T.M. Chin, A.S. Willsky, and W.C. Karl, "A New Highly Parallel, Iterative Algorithm for Square-Root Filtering and Smoothing," accepted for publication in *Automatica.*

16. A. S. Willsky, "Opportunities and Challenges in Signal Processing and Analysis," Proc. of INRIA 25th Anniversary Conf., Paris, France, Dec. 1992, Springer-Verlag, edited by A. Bensoussan).

17. T.M. Chin, M.R. Luettgen, W.C. Karl, and A.S. Willsky, "An Estimation-Theoretic Perspective on Image Processing and the Calculation of Optical Flow," in *Advances in Image Sequence Processing*, M. Sezan and R. Lagendijk, eds., Kluwer, 1993.

18. M. Basseville, A. Benveniste, K.C. Chou, S.A. Golden, R. Nikoukhah, and A.S. Willsky, "Modeling and Estimation of Multiresolution Stochastic Processes," *IEEE Transactions on Information Theory,* special issue on Wavelet Transforms and Multiresolution Signal Analysis, Vol. 38, No. 2, March 1992, pp. 766-784.

19. K.C. Chou, A.S. Willsky, and A. Benveniste, "Multiscale Recursive Estimation, Data Fusion, and Regularization," *IEEE Trans. on Automatic Control.*

20. M. Luettgen, K.C. Chou, , A.S. Willsky, and A. Benveniste, "Multiresolution Statistical Image Processing with Applications to Motion Estimation," 7th IEEE Workshop on Multidimensional Signal Processing, Lake Placid, NY, Sept. 1991.

21. K.C. Chou, A.S. Willsky, and R. Nikoukhah, "Multiscale Systems, Kalman Filters, and Riccati Equations," *IEEE Trans. on Automatic Control.*

22. M. Luettgen, W.C. Karl, A.S. Willsky, and R.R. Tenney, "Multiscale Representation of Markov Random Fields," special issue of the *IEEE Trans. on Signal Processing* on wavelet transforms.

23. M.R. Luettgen, W.C. Karl and A.S. Willsky, "Efficient Multiscale Regularization with Applications to the Computation of Optical Flow," *IEEE Trans. on Image Processing.*

24. M.R. Luettgen, W.C. Karl, and A.S. Willsky, "Multiresolution Methods in Statistical Analysis, "Proc. of the Int'l Wavelets and Applications Conf., Toulouse, France, June 1992; also an abstract of this will appear in Proc. SPIE Conf. on Intelligent Robots and Computer Vision XI: Biological Neural Networks and 3-D Methods, Boston, MA

25. E. Fabre, A. Benveniste, R. Nikoukhah, and A.S. Willsky. "Multiresolution Systems on Trees," Proc. of the Int'l Wavelets and Applications Conf., Toulouse, France, June 1992.

26. S.A. Golden and A.S. Willsky, "Identifying Multiscale Statistical Models Using the Wavelet Transform", to be submitted to the IEEE Transactions on Signal Processing.

27. K.C. Chou, S.A. Golden and A.S. Willsky, "Wavelet Transform-Based Algorithms for the Fusion of Multiresolution Data", Signal Processing.

28. A.S. Willsky, "Wavelet Transforms, Data Fusion, and Multiresolution Statistical Signal Processing", Keynote Address at the IEEE International Conference on Systems Engineering, Dayton, OH, Aug. 1991.

29. Chou, K.C. and A.S. Willsky, "Modeling and Estimation of Multiscale Processes", 25th Asitomar Conference, Monterey, CA, Nov. 1991.

30. M.R. Luettgen, "Multiresolution Models and Algorithms for Random Fields and Image Processing," Ph.D. thesis, May 1993.

31. Miller, E.L., Bhatia, M., Karl, W.C., and Willsky, A.S., "Statistical Methods in Multiresolution Signal Processing," IMA Conference on Multiscale Stochastic Processes Analysed Using Multifractals and Wavelets, Cambridge, England, March 1993.

32. M.R. Luettgen and A.S. Willsky, "Likelihood Calculation for Multiscale Image Models with Applications in Texture Discrimination," *IEEE Trans. on Image Processing.*

33. Irving, W.W., Daniel, M.M., Fieguth, P.W., Karl, W.C., and Willsky, A.S., "Efficient Algorithms for Gauss-Markov Random Fields," 8th IEEE Workshop on Multidimensional Signal Processing, Cannes, France, Sept. 1993.

34. M.R. Luettgen, W.C. Karl, A.S. Willsky, and R.R. Tenney, "Multiscale Representation of Markov Random Fields, with Applications to Texture Labeling," Int'l Conf. on Acoustics, Speech, and Sig. Proc., April, 1993.

35. M.R. Luettgen, W.C. Karl, and A.S. Willsky, "Multiscale Signal Processing" (abstract only), Amer. Math. Society Wavelets and Applications Conf., S. Hadley, MA, July 1992.

36. M.R. Luettgen, M. Bhatia, R.E. Learned, E.L. Miller, W.C. Karl, and A.S. Willsky, "Statistical Methods in Multiresolution Signal Processing," (abstract only) 1992 Digital Signal Processing Workshop, Starved Rock State Park, IL, Sept. 1992.

37. M.R. Luettgen A.S. Willsky, "Likelihood Calculation for a Class of Multiscale Stochastic Models," 32nd IEEE Conf. on Decision and Control, San Antonio, Texas, Dec. 1993.

38. Milanfar, P., Bhatia, M., Belcastro, L.A., Jaggi, S., Karl, W.C., and Willsky, A.S., "Geometric and Multiresolution Statistical Methods for Reconstruction from Projections," 8th IEEE Workshop on Multidimensional Signal Processing, Cannes, France, Sept. 1993.

39. P. Milanfar, W.C. Karl, and A.S. Willsky, "Statistical Approaches to the Tomographic Reconstruction of Finitely Parametrized Geometric Objects," SPIE Conf. on Neural and Stochastic Methods in Image and Signal Processing, San Diego, July 1992.

40. P. Milanfar, "Estimation and Reconstruction of Geometric Featues from Tomographic Data," Ph. D. thesis, MIT, June 1993.

41. P. Milanfar, W.C. Karl, S.R. Kulkarni, and A.S. Willsky, "Geometric Aspects of Tomographic Signal Processing", 7th IEEE Workshop on Multidimensional Signal Processing, Lake Placid, September 1991.

42. A.S. Lele, S.R. Kulkarni, and A.S. Willsky, "Convex Polygon Estimation from Support Line Measurements and Applications to Target Reconstruction from Laser Radar Data," *Journal of the Optical Society of America-A,* Vol. 9, No. 10, Oct. 1992.

43. Chou, K.C., "A Stochastic Modeling Approach to Multiscale Signal Processing," Ph. D. thesis, MIT Dept. of Elec. Eng. and Comp. Sci., May 1991.

44. Karl, W.C., Kulkarni, S.R., Verghese, G.C., and Willsky, A.S., "Local Tests for Consistency of Support Plane Data," submitted to *Mathematics of Image Processing and Computer Vision.*

45. S. Jaggi, "Dynamic Estimation of the Geometry of Objects in Two and Three Dimensions," S.M. Thesis, August 1992.

46. P. Milanfar, W.C. Karl, and A.S. Willsky, "A Geometric Framework for Tomographic Reconstruction," Proc. of the Johns Hopkins Conf., Baltimore, Md., March 1993.

47. Miller, E.L. and Willsky, A.S., "A Multiscale Approach to the Solution of Linear Inverse Problems," accepted for publication in *Journal on Applied and Computational Harmonic Analysis.*

48. Bhatia, M., Karl, W.C., and Willsky, A.S., "A Fast Multiscale Reconstruction Method from Complete but Possibly Noisy Projection Data," submitted to *IEEE Transactions on Medical Imaging.*

49. Bhatia, M., Karl, W.C., and Willsky, A.S., "Using Natural Wavelet Bases and Multiscale Stochastic Models for Tomographic Reconstruction," submitted to *IEEE Trans. on Image Processing.*

50. T.M. Chin, W.C. Karl, A.S. Willsky, and A.J. Mariano, "Square-Root Filtering in Time-Sequential Estimation of Random Fields," IS&T/SPIE 1993 International Symposium on Electronic Imaging: Science and Technology, San Jose, CA, Jan.31-Feb. 4 1993

51. J.L. Prince, and A.S. Willsky, "A Hierarchical Algorithm for Limited-Angle Tomographic Reconstruction", *IEEE Transactions on Image Processing,* Vol. 2, No. 3, July 1993, pp. 311-333.

52. W.C. Karl, G.C. Verghese, and A.S. Willsky, "Reconstructing Ellipsoids from Projections," accepted for publication in *CGVIP: Graphical Models and Image Processing.*

53. E.L. Miller and A.S. Willsky, "A Multiscale Approach to Solving One-Dimensional Inverse Problems," IEEE-PS International Symposium on Time-Frequency and Time-Scale Analysis, Vancouver, Oct. 1992.

54. Milanfar, P., Karl, W.C., and Willsky, A.S., "Divergence-Based Methods for Tomographic Reconstruction from Noisy Estimates of Object Moments," submitted to the *IEEE Trans. on Image Processing.*

55. Milanfar, P., Karl, W.C., and Willsky, A.S., "Estimation of Polygonal Objects from Tomographic Measurements," accepted for publication in *CGVIP: Graphical Models and Image Processing.*

56. Milanfar, P., Verghese, G.C., Willsky, A.S., and Karl, W.C., "Array Processing Techniques for Object Reconstruction from Moments with Applications in Tomography," accepted for publication in *IEEE Trans. on Signal Processing.*

57. Belcastro, L.A., Karl, W.C., and Willsky, A.S., "Tomographic Reconstruction of Polygons Based on Noisy Estimates of Projection Knots and Chord Lengths," submitted to *CGVIP: Graphical Models and Image Processing*.

58. Luettgen, M.R. and Willsky, A.S., "Multiscale Smoothing Error Models," accepted for publication in *IEEE Trans. on Automatic Control*.

59. Learned, R.E. and Willsky, A.S., "Wavelet Packet Based Transient Signal Classification," submitted to *J. on Applied and Computational Harmonic Analysis*.

60. Benveniste, A., Nikoukhah, R., and Willsky, A.S., "Multiscale System Theory," *IEEE Trans. on Circuits and Systems*.

61. Belcastro, L.A., "Tomographic Reconstruction of Polygons from Projection Knot and Chord Length Measurements, S.M. thesis, Sept. 1993.

62. Willsky, A.S., "Multiresolution Models for Random Fields and Their Use in Statistical Signal and Image Processing," plenary address, 8th IEEE Workshop on Multidimensional Signal Processing, Cannes, France, Sept. 1993.

63. Jaggi, S., Karl, W.C., and Willsky, A.S., " Dynamic Estimation of Left-Ventricular Ejection Fraction," SPIE/SPSE Symposium on Electronic Imaging, San Jose, CA, January 1993.

64. Jaggi, S., Karl, W.C., and Willsky, A.S., " A Kalman-Filtering-Based Procedure for the Estimation of Left-Ventricular Ejection Fraction," accepted for publication in the *IEEE Trans. on Medical Imaging*.

65. Bhatia, M., Karl, W.C., and Willsky, A.S., "A Multiscale Method for Tomographic Reconstruction," SPIE 1993 Int'l. Symposium on Optics, Imaging, and Instrumentation, San Diego, CA, July 1993.

66. Miller, E.L. and Willsky, A.S., "The Use of Wavelets and Multiresolution Stochastic Models for Linear Inverse Problems," 1994 SPIE Int'l Symposium on OE/Aerospace Sensing, Conference on Wavelet Applications III, Orlando, FL, April 1994.

67. Fieguth, P.W., Karl, W.C., Claus, B., and Willsky, A.S., "Multiresolution Statistical Estimation of North Pacific Ocean Height from Topex/Poseidon Satellite Altimetry," 1994 SPIE Int'l Symposium on Optics, Imaging and Instrumentation, San Diego, CA, July 1994.

68. M. Bhatia, "Multiscale Methods in Tomography," Ph.D. thesis, MIT, August 1994.

69. E. Miller and A.S. Willsky, "A Multiscale, Statistically-Based Inversion Scheme for the Linearized Inverse Scattering Problem," submitted to the *IEEE Trans. on Geoscience and Remote Sensing*.

70. E. Miller, "Multiscale Methods for Multichannel Inverse Problems," Ph.D. thesis, August 1994.

71. W.W. Irving, W.C. Karl, and A.S. Willsky, "The Use of Canonical Correlations for the Identification of Robust Multiscale Pyramidal Models of Random Fields," 1994 IEEE Conference on Decision and Control.

72. M. Bhatia, W.C. Karl, and A.S. Willsky, "Efficient Multiresolution Methods in Tomography," IEEE Conference on Image Processing, Austin, TX, Nov. 1994.

73. P. W. Fieguth, W.C. Karl, and A.S. Willsky, "Large-Scale Statistically Optimal Data Assimilation with Applications in Satellite-Based Remote Sensing," featured invited presentation at the IEEE Conference on Image Processing, Austin, TX, Nov. 1994.

74. P.W. Fieguth, W.C. Karl, A.S. Willsky, and C.Wunsch, "Multiresolution Statistical Estimation of North Pacific Ocean Height from Topex/Poseidon Satellite Altimetry," accepted for publication in the *IEEE Trans. on Geoscience and Remote Sensing*.

75. P.W. Fieguth, A.S. Willsky, and W.C. Karl, "Multiresolution Processing of Topex/Poseidon Satellite Altimetry Data," IEEE Oceans '94, Brest, France, Sept. 1994.

76. M. Bhatia, W.C. Karl, and A.S. Willsky, "Multiresolution Algebraic Reconstruction from Projections," in preparation for submission.

77. E.L. Miller and A.S. Willsky, "A Multiresolution Statistical Approach to Electromagnetic Imaging," IEEE Conference on Image Processing, Austin, Texas, Nov. 1994.

78. W.W. Irving, "Multiresolution Stochastic Modeling with Applications in Image Analysis and Synthetic Aperture Radar," Ph.D. thesis, MIT, to be completed Aug. 1995.

79. P.W. Fieguth, "Multiresolution Statistical Solutions to Large-Scale Data Assimilation and Image Processing Problems, with Applications in Oceanographic Data Analysis," Ph.D. thesis, MIT, to be completed Aug. 1995.

80. W.W. Irving A.S. Willsky, and L.M. Novak, "Statistical Discrimination Based on Multiresolution SAR Imagery," in preparation for submission to the *IEEE Trans. on Aerospace and Elec. Systems*.

81. E.L. Miller and A.S. Willsky, "Multiresolution Methods for Anomaly Detection, Localization, and Characterization from Electromagnetic Inverse Scattering Data," submitted to the special issue on multiresolution methods of the *Journal of Multidimensional Signals and Systems*.

82. E.L. Miller and A.S. Willsky, "Multiresolution Statistical Methods for Inverse Scattering Problems," invited paper for the URSI meeting, 1995.

83. E.L. Miller and A.S. Willsky, "An Efficient Multiresolution Algorithm for Statistically Regularized Nonlinear Solution to Inverse Scattering Problems," to be submitted to *Radio Science*.

84. M. Bhatia, W.C. Karl, and A.S. Willsky, "A Multiresolution Algorithm for Direct Texture Discrimination from Tomographic Data," in preparation for submission to *IEEE Trans. on Image Processing*.

85. P.W. Fieguth, W.C. Karl, and A.S. Willsky, "Multiresolution Algorithms for Surface Reconstruction," in preparation for submission to *Computer Graphics and Visual Image Processing*.

86. P.W. Fieguth, W.W. Irving, and A.S. Willsky, "Seamless Multiresolution Stochastic Modeling of Random Fields Using Overlapped Trees," in preparation for submission to the 1995 Int'l. Conf. on Image Processing.

87. P.W. Fieguth, A.S. Willsky, C. Wunsch, and W.C. Karl, "Efficient Multiresolution Mapping of Topex/Poseidon Satellite Altimetry Data with Applications in Oceanography and Geodesy," invited paper for the 1995 Int'l. Conf. on Acoustics, Speech, and Sig. Proc.

88. P.W. Fieguth, A.S. Willsky, and W.C. Karl, "Efficient Multiresolution Counterparts to Variational Approaches to Surface Reconstruction," submitted to the 1995 Int'l. Conf. on Computer Vision.

89. P. Milanfar, A.S. Willsky, G.C. Verghese, and W.C. Karl, "An Array Processing Approach to Reconstruction of Polygons from Moments," 1994 Int'l. Conf. on Image Processing, Austin, TX, Nov. 1994.

90. M. Bhatia, A.S. Willsky, and W.C. Karl, "A Direct Multiresolution Approach to Texture Discrimination from Noisy Projection Measurements," in preparation for submission.